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On complete convergence for arrays of rowwise weakly dependent random variables[☆]

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ABSTRACT

Some sufficient conditions for complete convergence for arrays of rowwise $\tilde{\rho}$-mixing random variables are presented without the assumption of identical distributions. As an application, the Marcinkiewicz–Zygmund type strong law of large numbers for weighted sums of $\tilde{\rho}$-mixing random variables is obtained.

1. Introduction

Let $\{X_i, i \geq 1\}$ be a sequence of independent observations from a population distribution. A common expression for these linear statistics is $T_n \doteq \sum_{i=1}^{n} a_{ni}X_i$, where the weights a_{ni} are either real constants or random variables independent of X_i. Using an observation on Bernstein's inequality by Cheng [1], Bai et al. [2] established an extension of the Hardy–Littlewood strong law for T_n. This complements a result of Cuzick [3, Theorem 2.2]. Recently, Bai and Cheng [4] showed extensions of the Marcinkiewicz–Zygmund strong law under certain moment conditions on both the weights and the distribution. These complement the results of Cuzick [3] and Bai et al. [2].

Inspired by Bai and Cheng [4], our main purpose in this work is to generalize the main result of Bai and Cheng [4] for independent and identically distributed random variables to the case of arrays of rowwise $\tilde{\rho}$-mixing random variables. We will study the complete convergence for arrays of rowwise $\tilde{\rho}$-mixing random variables. As an application, the Marcinkiewicz–Zygmund type strong law of large numbers for weighted sums of $\tilde{\rho}$-mixing sequences of random variables is obtained. The results presented in this work are obtained by using the truncated method and the classical maximal type inequality of $\tilde{\rho}$-mixing random variables (Lemma 1.1).

Let $\{X_n, n \geq 1\}$ be a random variable sequence defined on a fixed probability space $(\Omega, \mathcal{F}, P)$. Write $\mathcal{F}_S = \sigma(X_i, i \in S \subset \mathbb{N})$. Given σ-algebras $\mathcal{B}, \mathcal{R}$ in $\mathcal{F}$, let

$$\rho(\mathcal{B}, \mathcal{R}) = \sup_{X \in L_2(\mathcal{B}), Y \in L_2(\mathcal{R})} \frac{|EXY - EXEY|}{(\operatorname{Var} X \cdot \operatorname{Var} Y)^{1/2}}.$$

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Define the $\tilde{\rho}$-mixing coefficients by

$$\tilde{\rho}(k) = \sup\{\rho(\mathcal{F}_{\mathcal{S}}, \mathcal{F}_{\mathcal{T}}) : \text{ finite subsets } S, T \subset \mathbb{N}, \text{ such that } \mathrm{dist}(S, T) \geq k\}, \quad k \geq 0.$$

Definition 1.1. A sequence of random variables $\{X_n, n \geq 1\}$ is said to be a $\tilde{\rho}$-mixing sequence if there exists $k \in \mathbb{N}$ such that $\tilde{\rho}(k) < 1$.

An array of random variables $\{X_{ni}, i \geq 1, n \geq 1\}$ are called rowwise $\tilde{\rho}$-mixing random variables if for every $n \geq 1$, $\{X_{ni}, i \geq 1\}$ is a $\tilde{\rho}$-mixing sequence of random variables.

$\tilde{\rho}$-mixing random variables were introduced by Bradley [5] and many applications have been found. $\tilde{\rho}$-mixing is similar to ρ-mixing, but they are in some ways quite different. Many authors have studied this concept and provided interesting results and applications. See, for example, [5] for the central limit theorem, Bryc and Smolenski [6], Peligrad and Gut [7], and Utev and Peligrad [8] for moment inequalities, Gan [9], Kuczmaszewska [10], and Wu and Jiang [11] for almost sure convergence, Peligrad and Gut [7], Gan [9], Cai [12,13], Kuczmaszewska [14], Zhu [15], An and Yuan [16], and Sung [17] for complete convergence, Peligrad [18] for the invariance principle, Budsaba et al. [19,20] for limiting behavior of moving average processes based on a sequence of $\tilde{\rho}$ random variables, and so forth. When these are compared with the corresponding results for independent random variable sequences, there still remains much to be desired.

Definition 1.2. An array of random variables $\{X_{ni}, i \geq 1, n \geq 1\}$ is said to be stochastically dominated by a random variable X if there exists a positive constant C such that

$$P(|X_{ni}| > x) \leq CP(|X| > x) \tag{1.1}$$

for all $x \geq 0$, $i \geq 1$ and $n \geq 1$.

The following lemmas are useful for the proof of the main results.

Lemma 1.1 (*Utev and Peligrad, [8], Theorem 2.1]*). *Let $\{X_n, n \geq 1\}$ be a $\tilde{\rho}$-mixing sequence of random variables, with $EX_i = 0$, $E|X_i|^p < \infty$ for some $p \geq 2$ and for every $i \geq 1$. Then there exists a positive constant C depending only on p such that*

$$E\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} X_i\right|^p\right) \leq C\left\{\sum_{i=1}^{n} E|X_i|^p + \left(\sum_{i=1}^{n} EX_i^2\right)^{p/2}\right\}.$$

Lemma 1.2. *Let $\{X_n, n \geq 1\}$ be a sequence of random variables which is stochastically dominated by a random variable X. For any $\alpha > 0$ and $b > 0$,*

$$E|X_n|^{\alpha} I\left(|X_n| \leq b\right) \leq C_1\left[E|X|^{\alpha} I\left(|X| \leq b\right) + b^{\alpha} P\left(|X| > b\right)\right], \tag{1.2}$$

$$E|X_n|^{\alpha} I\left(|X_n| > b\right) \leq C_2 E|X|^{\alpha} I\left(|X| > b\right), \tag{1.3}$$

where C_1 and C_2 are positive constants.

2. The main results

Throughout the work, let $I(A)$ be the indicator function of the set A. C denotes a positive constant which may be different in various places and $a_n = O(b_n)$ stands for $a_n \leq Cb_n$. Our main results are as follows.

Theorem 2.1. *Let $\{X_{ni} : i \geq 1, n \geq 1\}$ be an array of rowwise $\tilde{\rho}$-mixing random variables which is stochastically dominated by a random variable X and $\{a_{ni} : i \geq 1, n \geq 1\}$ be an array of real numbers. Assume that there exist some δ with $0 < \delta < 1$ and some α with $0 < \alpha \leq 2$ such that $\sum_{i=1}^{n} |a_{ni}|^{\alpha} = O(n^{\delta})$, and assume further that $EX_{ni} = 0$ if $1 < \alpha \leq 2$. $p \geq 1/\alpha$. If for some $h > 0$ and $\gamma > 0$ such that $E\exp\left(h|X|^{\gamma}\right) < \infty$, then*

$$\sum_{n=1}^{\infty} n^{p\alpha-2} P\left(\max_{1\leq j\leq n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon b_n\right) < \infty, \quad \forall\, \varepsilon > 0, \tag{2.1}$$

where $b_n \doteq n^{1/\alpha}\log^{1/\gamma} n$.

Proof. For fixed $n \geq 1$, define

$$X_i^{(n)} = X_{ni} I(|X_{ni}| \leq b_n), \quad i \geq 1, \qquad T_j^{(n)} = \sum_{i=1}^{j} a_{ni}\left(X_i^{(n)} - EX_i^{(n)}\right), \quad j = 1, 2, \ldots, n.$$

It is easy to check that for any $\varepsilon > 0$,

$$P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon b_n\right) \le \sum_{i=1}^{n} P\left(|X_{ni}| > b_n\right) + P\left(\max_{1\le j\le n}\left|T_j^{(n)}\right| > \varepsilon b_n - \max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}EX_i^{(n)}\right|\right). \tag{2.2}$$

Firstly, we will show that

$$b_n^{-1}\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}EX_i^{(n)}\right| \to 0, \quad \text{as } n\to\infty. \tag{2.3}$$

By $\sum_{i=1}^{n}|a_{ni}|^{\alpha} = O(n)$ and Hölder's inequality, we have for $1 \le k < \alpha$ that

$$\sum_{i=1}^{n}|a_{ni}|^{k} \le \left(\sum_{i=1}^{n}\left(|a_{ni}|^{k}\right)^{\frac{\alpha}{k}}\right)^{\frac{k}{\alpha}}\left(\sum_{i=1}^{n}1\right)^{\frac{\alpha-k}{\alpha}} \le Cn. \tag{2.4}$$

Hence, when $1 < \alpha \le 2$, we have by $EX_{ni} = 0$, (1.3) of Lemma 1.2 and (2.4) that

$$\begin{aligned} b_n^{-1}\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}EX_i^{(n)}\right| &\le b_n^{-1}\sum_{i=1}^{n}|a_{ni}|E|X_{ni}|I(|X_{ni}| > b_n) \\ &\le Cb_n^{-1}n\sum_{k=n}^{\infty}E|X|I(b_k < |X| \le b_{k+1}) \le Cb_n^{-1}n\sum_{k=n}^{\infty}b_{k+1}P(|X| > b_k) \\ &\le Cb_n^{-1}n\sum_{k=n}^{\infty}b_{k+1}\frac{E\exp(h|X|^{\gamma})}{\exp(hb_k^{\gamma})} \le Cb_n^{-1}\sum_{k=n}^{\infty}(k+1)^{1/\alpha+1}(\log(k+1))^{1/\gamma}k^{-hk^{\gamma/\alpha}} \\ &\le Cn^{-1/\alpha}(\log n)^{-1/\gamma} \to 0, \quad \text{as } n\to\infty. \end{aligned} \tag{2.5}$$

The elementary Jensen inequality implies that for any $0 < s < t$,

$$\left(\sum_{i=1}^{n}|a_{ni}|^{t}\right)^{1/t} \le \left(\sum_{i=1}^{n}|a_{ni}|^{s}\right)^{1/s}. \tag{2.6}$$

Therefore, when $0 < \alpha \le 1$, we have by (1.2) of Lemma 1.2 and (2.6) that

$$\begin{aligned} b_n^{-1}\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}EX_i^{(n)}\right| &\le Cb_n^{-1}\sum_{i=1}^{n}|a_{ni}|\left(E|X|I(|X| \le b_n) + b_nP(|X| > b_n)\right) \\ &\le Cb_n^{-1}n^{\delta/\alpha}\sum_{k=2}^{n}E|X|I(b_{k-1} < |X| \le b_k) + \frac{Cn^{\delta/\alpha}E\exp(h|X|^{\gamma})}{\exp(hb_n^{\gamma})} \\ &\le Cb_n^{-1}n^{\delta/\alpha}\sum_{k=2}^{n}b_k\frac{E\exp(h|X|^{\gamma})}{\exp(hb_{k-1}^{\gamma})} + \frac{Cn^{\delta/\alpha}}{n^{hn^{\gamma/\alpha}}} \\ &\le C(\log n)^{-1/\gamma}n^{\delta/\alpha-1/\alpha} + \frac{Cn^{\delta/\alpha}}{n^{hn^{\gamma/\alpha}}} \to 0, \quad \text{as } n\to\infty. \end{aligned} \tag{2.7}$$

(2.5) and (2.7) yield (2.3). Hence, for n large enough,

$$P\left(\max_{1\le j\le n}\left|\sum_{i=1}^{j} a_{ni}X_{ni}\right| > \varepsilon b_n\right) \le \sum_{i=1}^{n} P\left(|X_{ni}| > b_n\right) + P\left(\max_{1\le j\le n}\left|T_j^{(n)}\right| > \frac{\varepsilon}{2}b_n\right).$$

To prove (2.1), we only need to show that

$$I \doteq \sum_{n=1}^{\infty}n^{p\alpha-2}\sum_{i=1}^{n}P\left(|X_{ni}| > b_n\right) < \infty, \tag{2.8}$$

$$J \doteq \sum_{n=1}^{\infty}n^{p\alpha-2}P\left(\max_{1\le j\le n}\left|T_j^{(n)}\right| > \frac{\varepsilon}{2}b_n\right) < \infty. \tag{2.9}$$

By Definition 1.2, we can see that

$$I \le C\sum_{n=1}^{\infty} n^{p\alpha-1}\frac{E\exp(h|X|^{\gamma})}{\exp(hb_n^{\gamma})} \le C\sum_{n=1}^{\infty}\frac{n^{p\alpha-1}}{n^{hn^{\gamma/\alpha}}} < \infty. \tag{2.10}$$

For $q \ge 2$, it follows from Lemma 1.1, the C_r inequality and Jensen's inequality that

$$\begin{aligned} J &\le C\sum_{n=2}^{\infty} n^{p\alpha-2}b_n^{-q}E\left(\max_{1\le j\le n}\left|T_j^{(n)}\right|^q\right) \\ &\le C\sum_{n=2}^{\infty} n^{p\alpha-2}b_n^{-q}\sum_{i=1}^{n}|a_{ni}|^q E\left|X_i^{(n)}\right|^q + C\sum_{n=2}^{\infty} n^{p\alpha-2}b_n^{-q}\left(\sum_{i=1}^{n}|a_{ni}|^2 E\left|X_i^{(n)}\right|^2\right)^{q/2} \\ &\doteq J_1 + J_2. \end{aligned} \tag{2.11}$$

Taking $q > \max\{2, \alpha(p\alpha-1)/(1-\delta)\}$, which implies that $q > \alpha$, it follows from (1.2) of Lemma 1.2 and (2.6) that

$$\begin{aligned} J_1 &= C\sum_{n=2}^{\infty} n^{p\alpha-2}b_n^{-q}\sum_{i=1}^{n}|a_{ni}|^q E\,|X_{ni}|^q I(|X_{ni}| \le b_n) \\ &\le C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}b_n^{-q}\sum_{k=2}^{n}E|X|^q I(b_{k-1} < |X| \le b_k) + C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}\frac{E\exp(h|X|^{\gamma})}{\exp(hb_n^{\gamma})} \\ &\le C\sum_{k=2}^{\infty}\sum_{n=k}^{\infty} n^{p\alpha-2+q\delta/\alpha}n^{-q/\alpha}(\log n)^{-q/\gamma}b_k^q P(|X| > b_{k-1}) + C\sum_{n=2}^{\infty}\frac{n^{p\alpha-2+q\delta/\alpha}}{n^{hn^{\gamma/\alpha}}} \\ &\le C\sum_{k=2}^{\infty}\frac{k^{q/\alpha}(\log k)^{q/\gamma}}{(k-1)^{h(k-1)^{\gamma/\alpha}}} + C\sum_{n=2}^{\infty}\frac{n^{p\alpha-2+q\delta/\alpha}}{n^{hn^{\gamma/\alpha}}} < \infty. \end{aligned} \tag{2.12}$$

By (1.2) of Lemma 1.2 again, (2.6) and the C_r inequality, we can get that

$$\begin{aligned} J_2 &\le C\sum_{n=2}^{\infty} n^{p\alpha-2}b_n^{-q}\left[\sum_{i=1}^{n}|a_{ni}|^2\left[EX^2 I(|X| \le b_n) + b_n^2 P(|X| > b_n)\right]\right]^{q/2} \\ &\le C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}b_n^{-q}\left[EX^2 I(|X| \le b_n)\right]^{q/2} + C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}\left[P(|X| > b_n)\right]^{q/2} \\ &\le C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}b_n^{-q}E|X|^q I(|X| \le b_n) + C\sum_{n=2}^{\infty} n^{p\alpha-2+q\delta/\alpha}P(|X| > b_n) \\ &< \infty. \quad (\text{by } (2.12)) \end{aligned} \tag{2.13}$$

Therefore, the desired result (2.1) follows from (2.10)–(2.13) immediately. □

The following result provides the Marcinkiewicz–Zygmund type strong law of large numbers for weighted sums $\sum_{i=1}^{n} a_i X_i$ of $\tilde{\rho}$-mixing sequences of random variables.

Theorem 2.2. *Let $\{X_n, n \ge 1\}$ be a sequence of $\tilde{\rho}$-mixing random variables which is stochastically dominated by a random variable X and $\{a_n, n \ge 1\}$ be a sequence of real numbers. Assume that there exist some δ with $0 < \delta < 1$ and some α with $0 < \alpha \le 2$ such that $\sum_{i=1}^{n}|a_i|^{\alpha} = O(n^{\delta})$ and assume further that $EX_n = 0$ if $1 < \alpha \le 2$. $p \ge 1/\alpha$. If some $h > 0$ and $\gamma > 0$ are such that $E\exp(h|X|^{\gamma}) < \infty$, then for any $\varepsilon > 0$,*

$$\sum_{n=1}^{\infty} n^{p\alpha-2}P\left(\max_{1\le j\le n}|S_j| > \varepsilon b_n\right) < \infty \tag{2.14}$$

and

$$\lim_{n\to\infty}\frac{|S_n|}{b_n} = 0 \quad a.s., \tag{2.15}$$

where $b_n \doteq n^{1/\alpha}\log^{1/\gamma} n$ and $S_n = \sum_{i=1}^{n} a_i X_i$ for $n \ge 1$.

Proof. Like in the proof of Theorem 2.1, we can get (2.14), which yields that

$$\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\le j\le n} \left|S_j\right| > \varepsilon b_n\right) < \infty. \tag{2.16}$$

Therefore,

$$\begin{aligned}
\infty &> \sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\le j\le n} \left|S_j\right| > \varepsilon b_n\right) \\
&= \sum_{i=0}^{\infty} \sum_{n=2^i}^{2^{i+1}-1} n^{-1} P\left(\max_{1\le j\le n} |S_j| > \varepsilon n^{\frac{1}{\alpha}} (\log n)^{\frac{1}{\gamma}}\right) \\
&\ge \frac{1}{2} \sum_{i=1}^{\infty} P\left(\max_{1\le j\le 2^i} |S_j| > \varepsilon 2^{\frac{i+1}{\alpha}} (\log 2^{i+1})^{\frac{1}{\gamma}}\right).
\end{aligned}$$

By the Borel–Cantelli Lemma, we obtain that

$$\lim_{i\to\infty} \frac{\max_{1\le j\le 2^i} |S_j|}{2^{\frac{i+1}{\alpha}} (\log 2^{i+1})^{\frac{1}{\gamma}}} = 0 \quad \text{a.s.} \tag{2.17}$$

For all positive integers n, there exists a positive integer i_0 such that $2^{i_0-1} \le n < 2^{i_0}$. We have by (2.17) that

$$\frac{|S_n|}{b_n} \le \max_{2^{i_0-1}\le n<2^{i_0}} \frac{|S_n|}{b_n} \le \frac{2^{\frac{2}{\alpha}} \max_{1\le j\le 2^i} |S_j|}{2^{\frac{i_0+1}{\alpha}} (\log 2^{i_0+1})^{\frac{1}{\gamma}}} \left(\frac{i_0+1}{i_0-1}\right)^{\frac{1}{\gamma}} \to 0 \quad \text{a.s.,} \quad \text{as } i_0 \to \infty,$$

which implies (2.15). This completes the proof of the theorem. □

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